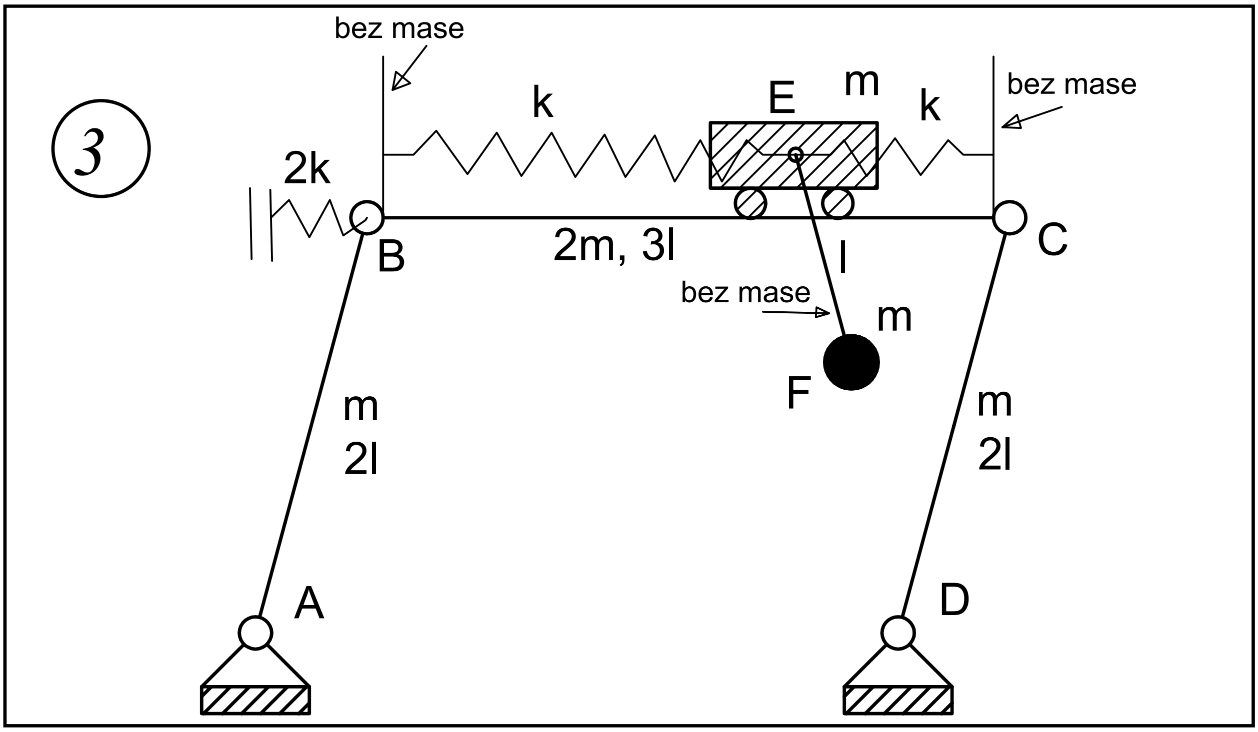
ГРАЂЕВИНСКИ ФАКУЛТЕТ УНИВЕРЗИТЕТА У БЕОГРАДУ
Катедра за Техничку механику и теорију конструкција
Писмени део испита из **ТЕХНИЧКЕ МЕХАНИКЕ - 2** **18.04.2007.**

ГРУПА **A**

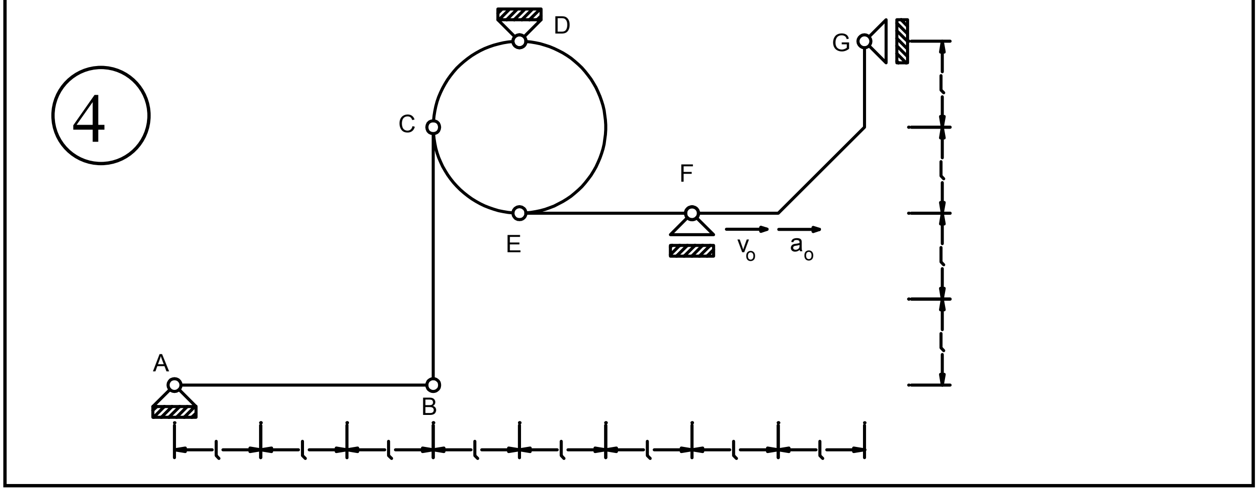
3. ЗАДАТАК: Механички систем на скици (1) креће се у вертикалној равни. Опруге су ненапрегнуте када је штап AB вертикалан,а колица масе m се налазе на средини штапа BC. Одредити:

- број степени слободе
- брзине средишта маса и угаоне брзине свих тела система, са скицом брзина карактерист. тачака
- кинетичку енергију система
- генералисане координате



4. ЗАДАТАК: У приказаном положају механизма на слици, познати су брзина тачке и убрзање тачке B : $v_B = v_0$, $a_B = a_0$. Одредити:

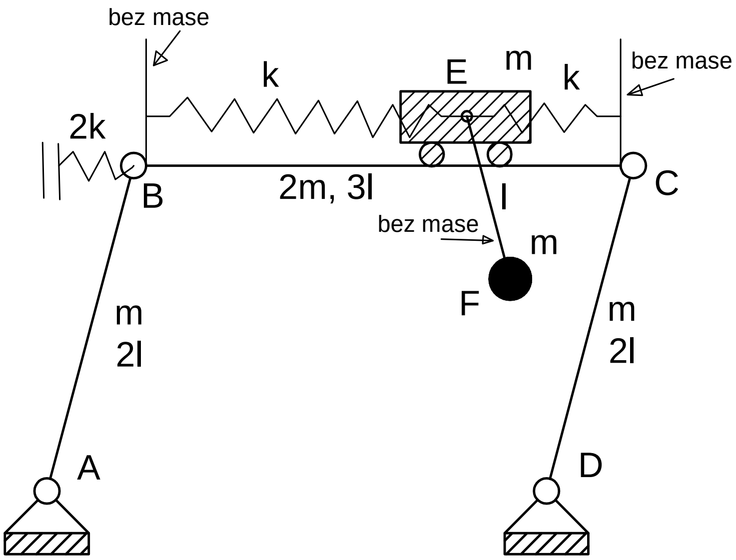
- Угаоне брзине свих тела система и брзине тачака A,C,E и G.
- Угаона убрзања свих тела система и убрзања тачака C и G.



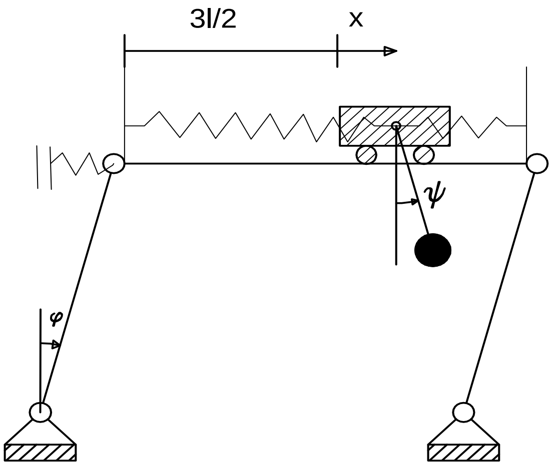
Напомена: Услов за полагање испита је:

- мин.20% (од 40%) на 1. и 2. задатку,
- мин.30% (од 60%) на 3. и 4. задатку.

Zadatak 1.

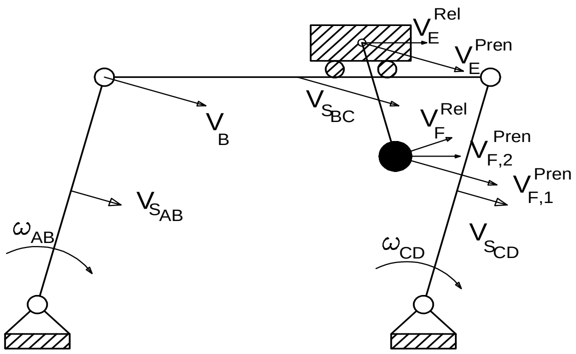


- **Broj stepeni slobode:**



$$
\begin{aligned}
n &= 3 \\
q_1 &= \varphi \\
q_2 &= x \\
q_3 &= \psi
\end{aligned}
$$

- **Brzine središta masa i ugaone brzine tela:**



- **Štap AB:**

$$\begin{aligned}\omega_{AB} &= \dot{\varphi}\\ v_{S_{AB}} &= \dot{\varphi}\cdot\ l\end{aligned}$$

- **Štap BC:**

$$\begin{aligned}\omega_{BC} &= 0\\ v_{S_{BC}} &= v_B = \dot{\varphi}\cdot\ 2l\end{aligned}$$

- **Štap CD:**

$$\begin{aligned}\omega_{CD} &= \omega_{AB} = \dot{\varphi}\\ v_{S_{CD}} &= v_{S_{AB}} = \dot{\varphi}\cdot\ l\end{aligned}$$

- **Kolica:**

$$\begin{aligned}v_E{}^{pren} &= v_{BC} = \dot{\varphi}\cdot 2l\\ v_E^{rel} &= \dot{x}\\ v_{E,x}^{pren} &= 2l\dot{\varphi}\cos\varphi\\ v_{E,y}^{pren} &= 2l\dot{\varphi}\sin\varphi\\ \vec{v_E} &= (2l\dot{\varphi}\cos\varphi + \dot{x})\vec{i} - 2l\dot{\varphi}\sin\varphi\vec{j}\end{aligned}$$

- **Tačka F:**

$$\begin{aligned}v_{F,1}^{pren} &= v_E^{pren} = \dot{\varphi}\cdot 2l\\ v_{F,2}^{pren} &= v_E^{rel} = \dot{x}\\ v_{F,1}^{pren,x} &= \dot{\varphi}\cdot 2l\cos\varphi\\ v_{F,1}^{pren,y} &= \dot{\varphi}\cdot 2l\sin\varphi\\ v_F^{rel,x} &= \dot{\psi}\cdot l\cos\psi\\ v_F^{rel,y} &= \dot{\psi}\cdot l\sin\psi\\ \vec{v_F} &= (\dot{x} + \dot{\psi}l\cos\psi + \dot{\varphi}2l\cos\varphi)\vec{i} + (\dot{\psi}l\sin\psi - \dot{\varphi}2l\sin\varphi)\vec{j}\end{aligned}$$

- **Kinetička energija:**

$$\begin{aligned}T_{AB} &= \frac{1}{2}m\cdot v_{S_{AB}}^2 + \frac{1}{2}I_{S_{AB}}\omega_{AB}^2\\ &= \frac{1}{2}m\dot{\varphi}^2l^2 + \frac{1}{2}\frac{1}{12}m(2l)^2\dot{\varphi}^2\\ &= = \frac{2}{3}ml^2\dot{\varphi}^2\end{aligned}$$

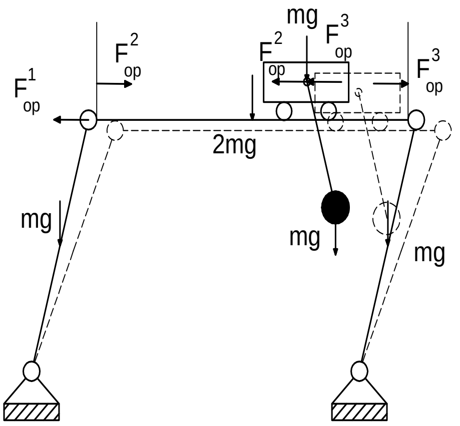
$$
\begin{aligned}
T_{CD} &= T_{AB} = \frac{2}{3}ml^2\dot{\varphi}^2 \\
T_{BC} &= \frac{1}{2}2m\cdot v_{S_{BC}}^2 + \frac{1}{2}I_{S_{BC}}\omega_{BC}^2 = \frac{1}{2}2m(\dot{\varphi}2l)^2 \\
&= 4ml^2\dot{\varphi}^2 \\
T_{Kolica} &= \frac{1}{2}m\cdot v_E^2 = \frac{1}{2}m[(2l\dot{\varphi}\cos\varphi+\dot{x})^2 + 2l\dot{\varphi}\sin\varphi^2] \\
&= \frac{1}{2}m\dot{x}^2 + 2m\dot{x}l\dot{\varphi}\cos\varphi + 2ml^2\dot{\varphi}^2 \\
T_F &= \frac{1}{2}m\dot{v}_E^2 = \frac{1}{2}m[(\dot{x}+\dot{\psi}l\cos\psi+\dot{\varphi}2l\cos\varphi)^2 + (\dot{\psi}l\sin\psi-\dot{\varphi}2l\sin\varphi)^2] \\
&= \frac{1}{2}m\dot{x}^2 + \frac{1}{2}m\dot{\psi}^2l^2 + 2m\dot{\varphi}^2l^2 + m\dot{x}\dot{\psi}l\cos\psi + 2m\dot{x}\dot{\varphi}l\cos\varphi + 2m\dot{\psi}\dot{\varphi}l^2 cos(\psi-\varphi) \\
T &= T_{AB} + T_{BC} + T_{CD} + T_{Kolica} + T_F \\
&= m\dot{x}^2 + \frac{28}{3}ml^2\dot{\varphi}^2 + \frac{1}{2}m\dot{\psi}^2l^2 + 4m\dot{x}l\dot{\varphi}\cos\varphi + m\dot{x}\dot{\psi}l\cos\psi + 2m\dot{\psi}\dot{\varphi}l^2\cos(\psi-\varphi)
\end{aligned}
$$

- **Generalisane sile:**
  - **opruga krutosti 2k:**

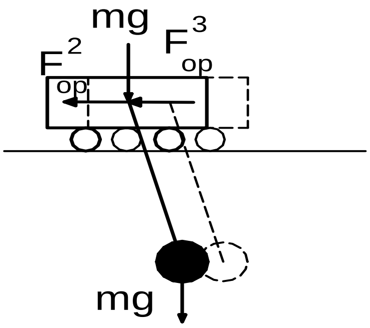
$$
\begin{aligned}
\Delta &= 2l\sin\varphi \\
F_{op}^1 &= 2k2l\sin\varphi
\end{aligned}
$$

  - **opruge krutosti k:**

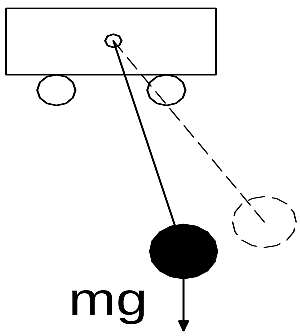
$$
\begin{aligned}
\Delta &= x \\
F_{op}^2 &= kx \\
F_{op}^3 &= kx
\end{aligned}
$$



$$
\begin{aligned}
Q_\varphi &= ?(\delta\varphi \neq 0, \delta x = 0, \delta\psi = 0) \\
\delta A &= 2mg\cdot(\delta\varphi\cdot l\sin\varphi) \\
&- 2k2l\sin\varphi\cdot(\delta\varphi\cdot 2l\cos\varphi) \\
&+ (2mg+mg+mg)\cdot(\delta\varphi\cdot 2l\sin\varphi) \\
&= Q_\varphi\delta\varphi \\
Q_\varphi &= 10mgl\sin\varphi - 8kl^2\sin\varphi\cos\varphi
\end{aligned}
$$

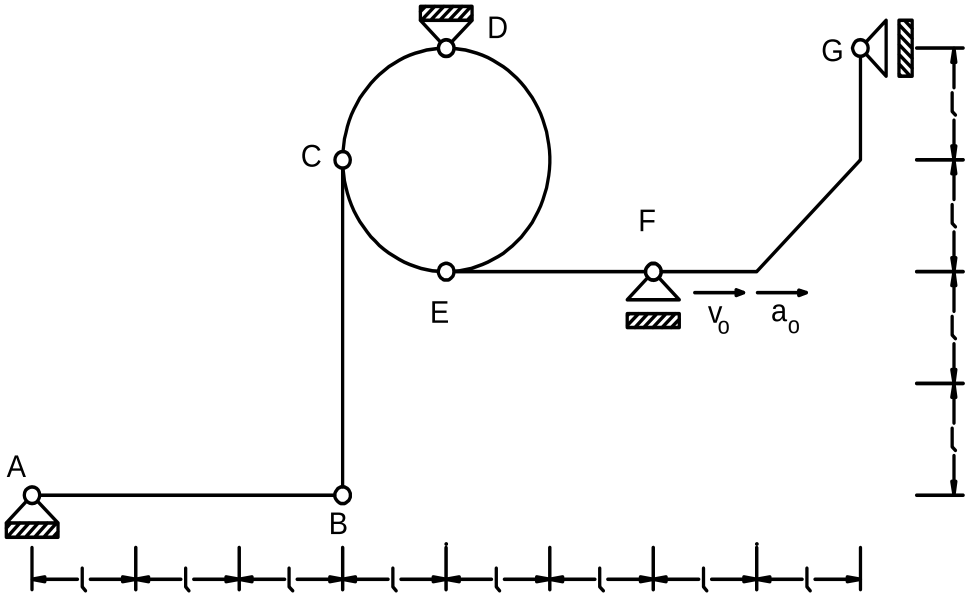


$$
\begin{aligned}
Q_x &= ?(\delta\varphi = 0, \delta x \neq 0, \delta\psi = 0) \\
\delta A &= -F^2_{op} \cdot \delta x - F^3_{op} \cdot \delta x \\
&= Q_x \delta x \\
Q_x &= -2kx
\end{aligned}
$$



$$
\begin{aligned}
Q_\psi &= ?(\delta\varphi = 0, \delta x = 0, \delta\psi \neq 0) \\
\delta A &= -mg \cdot \delta\psi \cdot l \sin\psi = Q_\psi \delta\psi \\
Q_\psi &= -mg \cdot l \sin\psi
\end{aligned}
$$

Zadatak 2.

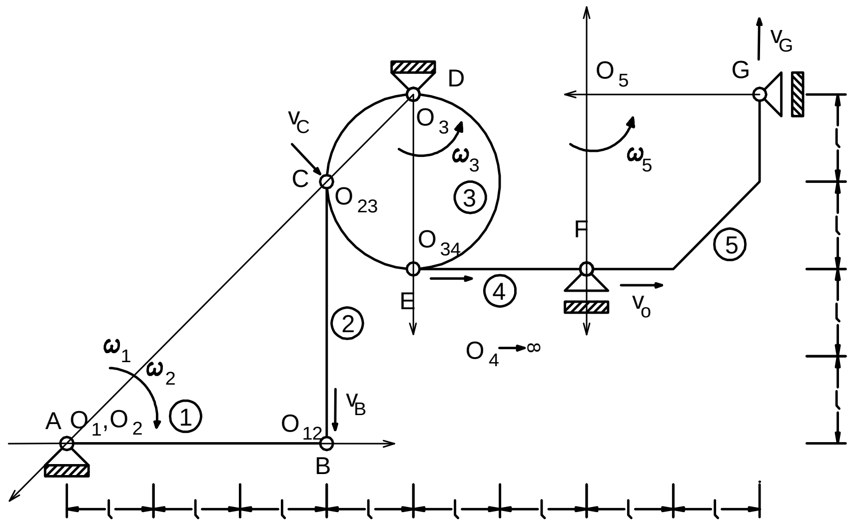


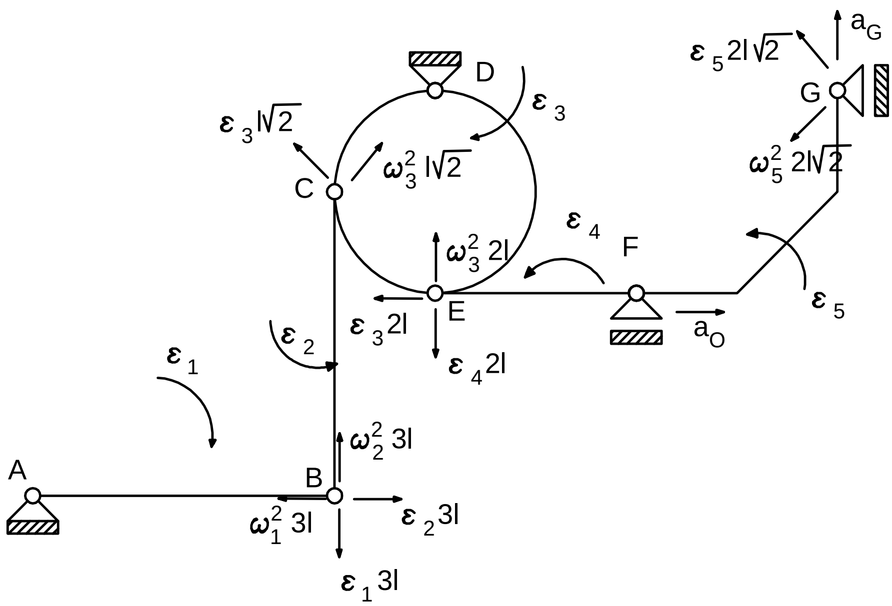
*** Brzine:**

$$
\begin{aligned}
v_F &= v_o = \omega_5 \cdot 0_5\bar{F} \Rightarrow \omega_5 = \frac{v_0}{2l} \\
v_G &= \omega_5 \cdot 0_5\bar{G} \Rightarrow v_G = v_0 \\
O_4 &\to \infty \Rightarrow \omega_4 = 0 \Rightarrow v_E = v_F = v_o \\
v_E &= \omega_3 \cdot 0_3\bar{E} \Rightarrow \omega_3 = \frac{v_0}{2l} \\
v_C &= \omega_3 \cdot 0_3\bar{C} \Rightarrow v_C = \frac{v_0}{2} \cdot \sqrt{2} \\
v_C &= \omega_2 \cdot 0_2\bar{C} \Rightarrow \omega_2 = \frac{v_0}{6l} \\
v_B &= \omega_2 \cdot 0_2\bar{B} \Rightarrow v_B = \frac{v_0}{2} \\
v_B &= \omega_1 \cdot 0_1\bar{B} \Rightarrow \omega_1 = \frac{v_0}{6l}
\end{aligned}
$$

*** Ubrzanja:**

$$
\vec{a_G} = \vec{a}_F + \vec{a}_{G,N}^F + \vec{a}_{G,T}^F
$$


v_G
G
O 5
D
v_C
O 3
ω3
ω5
C
O 23
3
F
O 34
5
4
E
v_o
2
O 4 →∞
ω1
ω2
A O 1, O 2
1
v_B
O 12
B


a_G
ε 5 2l√2
D
G
ε 3
ε 3 l√2
ω3² l√2
ω5² 2l√2
C
ε 4
F
ω3² 2l
ε 5
E
ε 3 2l
a_O
ε 2
ε 4 2l
ε 1
ω2² 3l
A
B
ω1² 3l
ε 2 3l
ε 1 3l

$$\begin{aligned}
X &: \quad 0 = a_o - (\frac{v_o}{2l})^2 \cdot 2l\sqrt{2} \cdot \frac{\sqrt{2}}{2} - \varepsilon_5 \cdot 2l\sqrt{2}\frac{\sqrt{2}}{2} \Rightarrow \varepsilon_5 = \frac{a_o}{2l} - \frac{v_o^2}{4l^2} \\
Y &: \quad a_G = -(\frac{v_o}{2l})^2 \cdot 2l\sqrt{2} \cdot \frac{\sqrt{2}}{2} + \varepsilon_5 \cdot 2l\sqrt{2}\frac{\sqrt{2}}{2} \Rightarrow a_G = a_o - \frac{v_o^2}{l} \\
\vec{a_E} &= \vec{a}_F + \vec{a}_{E,N}^F + \vec{a}_{E,T}^F \\
\vec{a_E} &= \vec{a}_D + \vec{a}_{E,N}^D + \vec{a}_{E,T}^D \\
X &: \quad a_o = -\varepsilon_3 \cdot 2l \Rightarrow \varepsilon_3 = -\frac{a_o}{2l} \\
Y &: \quad -\varepsilon_4 \cdot 2l = (\frac{v_o}{2l})^2 \cdot 2l \Rightarrow \varepsilon_4 = -\frac{v_o^2}{4l^2} \\
\vec{a_C} &= \vec{a}_D + \vec{a}_{C,N}^D + \vec{a}_{C,T}^D \\
X &: \quad a_{Cx} = -(\frac{v_o}{2l})^2 \cdot l\sqrt{2} \cdot \frac{\sqrt{2}}{2} - \varepsilon_3 \cdot l\sqrt{2}\frac{\sqrt{2}}{2} \Rightarrow a_{Cx} = \frac{v_o^2}{4l} + \frac{a_o}{2} \\
Y &: \quad a_{Cy} = (\frac{v_o}{2l})^2 \cdot l\sqrt{2} \cdot \frac{\sqrt{2}}{2} + \varepsilon_3 \cdot l\sqrt{2}\frac{\sqrt{2}}{2} \Rightarrow a_{Cy} = \frac{v_o^2}{4l} - \frac{a_o}{2} \\
\vec{a_B} &= \vec{a}_C + \vec{a}_{B,N}^C + \vec{a}_{B,T}^C \\
\vec{a_B} &= \vec{a}_A + \vec{a}_{B,N}^A + \vec{a}_{B,T}^A \\
X &: \quad \frac{v_o^2}{4l} + \frac{a_o}{2} + \varepsilon_2 \cdot 3l = -\omega_1^2 \cdot 3l \Rightarrow \varepsilon_2 = -\frac{a_o}{6l} - \frac{v_o^2}{9l^2} \\
Y &: \quad \frac{v_o^2}{4l} - \frac{a_o}{2} + (\frac{v_o}{6l})^2 \cdot 3l = -\varepsilon_1 \cdot 3l \Rightarrow \varepsilon_1 = \frac{a_o}{6l} - \frac{v_o^2}{9l^2}
\end{aligned}$$